

# NUMERICAL SIMULATION OF AIRCRAFT TRAILING VORTICES

Fred H. Proctor  
*NASA Langley Research Center*  
*Hampton Virginia 23681-2199*

and

George F. Switzer  
*Research Triangle Institute*  
*Hampton, Virginia 23666*

Paper: 7.12, pages 511-516  
Preprints of 9<sup>th</sup> Conference on Aviation, Range and Aerospace Meteorology  
11-15 September 2000, Orlando Florida  
American Meteorology Society

Fred H. Proctor<sup>\*</sup>  
 NASA Langley Research Center, Hampton Virginia  
 and  
 George F. Switzer  
 Research Triangle Institute, Hampton, Virginia

## 1. INTRODUCTION

The increase in air traffic is currently outpacing the development of new airport runways. This is leading to greater air traffic congestion, resulting in costly delays and cancellations. The National Aeronautics and Space Administration (NASA) under its Terminal Area Productivity (TAP) program is investigating new technologies that will allow increased airport capacity while maintaining the present standards for safety (Hinton 1995, Perry *et al* 1997). As an element of this program, the Aircraft Vortex Spacing System (AVOSS) is being demonstrated in July 2000, at Dallas Ft-Worth Airport (Hinton *et al* 2000). This system allows reduced aircraft separations, thus increasing the arrival and departure rates, while insuring that wake vortices from a leading aircraft do not endanger trailing aircraft. The system uses predictions of wake vortex position and strength based on input from the current weather state. This prediction is accomplished by a semi-empirical model developed from theory, field observations, and relationships derived from numerical wake vortex simulations (e.g., Sarpkaya 2000, Sarpkaya *et al* 2000). Numerical experiments with a Large Eddy Simulation (LES) model are being conducted in order to provide guidance for the enhancement of these prediction algorithms.

The LES simulations of wake vortices are carried out with NASA's Terminal Area Simulation System (TASS). Previous wake vortex investigations with TASS are described in Proctor (1996, 1998), Proctor *et al* (1997, 2000), Proctor and Han (1999), Shen *et al* (1999), Switzer and Proctor (2000), and Han *et al* (2000a, 2000b). The primary objective of these numerical studies has been to quantify vortex transport and decay in relation to atmospheric variables.

This paper summarizes many of the previous investigations with the TASS model and presents some new results regarding the onset of wake vortex decay.

## 2. VORTEX TRANSPORT

The lifting surfaces of an aircraft generate vorticity, which then rolls up into two dominant counter-rotating trailing vortices. The roll-up process is usually completed

within a distance equivalent to several wingspans. The separation, and circulation of the wake vortex pair immediately following roll up, is defined from conventional assumptions for an elliptically loaded wing as:  $b_0 = \pi B/4$  (see Fig. 1) and  $\Gamma_0 = \pm 4Mg/(\pi B\rho V_a)$ , where  $M$  is the mass of the aircraft,  $\rho$  is air density,  $g$  is acceleration due to gravity, and  $V_a$  is air speed.

The wake vortex system initially sinks at  $v_0 = \Gamma_0/(2\pi b_0)$  due to the mutual interaction of the vortices. The sink rate of the vortices usually decreases with time from the decay of circulation (e.g., Greene 1986, Sarpkaya 1998, Han *et al* 2000b). However, vertical air currents associated with turbulence (Han *et al* 2000c) and nonlinear vertical shear of the crosswind component (Proctor *et al* 1997, Proctor 1998) may cause both or one of the wake vortices to stall or rise. Strong stratification generates opposite sign vorticity, which weakens the circulation and diminishes the descent rate (e.g., Switzer and Proctor 2000). The lateral motion of wake vortices is generally determined by the crosswind in which the vortices are embedded. However, near the ground the wake vortex pair may diverge apart and rebound upward (e.g., Proctor and Han 1999).

## 3. VORTEX HAZARD

An inadvertent encounter with a leading aircraft's wake vortex can be dangerous and should be prevented (Perry *et al* 1997). The hazard to aircraft is primarily due to rolling moments, which can exceed the roll capability of the encountering aircraft. Although strong velocities may be associated with the small core of a wake vortex (about 1 m diameter), tangential velocities at larger radii are more influential in transferring rolling moments to an encountering aircraft. In order to assess the hazard of a wake vortex, an appropriate characterization must be defined. A representative characterization found by Hinton and Tatnal (1997) is the 5-15 m average circulation:

$$\bar{\Gamma}_{a,c} = \frac{\int_a^c \Gamma dr}{\int_a^c dr}$$

<sup>\*</sup> Corresponding author address: Fred H. Proctor, Airborne Systems Competency, NASA Langley Research Center, MS 156A, Hampton VA 23681-2199, email: [f.h.proctor@larc.nasa.gov](mailto:f.h.proctor@larc.nasa.gov)

where  $a=5\text{ m}$ , and  $c = 15\text{ m}$ . Prediction of this quantity is necessary in order to determine when a vortex no longer poses a threat to an encountering aircraft.

Wake vortex decay predictions are usually defined in terms of nondimensional parameters for time,  $t$ , eddy dissipation rate,  $\epsilon$ , and Brunt Vaisala frequency  $N$ , as:

$$T = tV_0/b_0 \quad \epsilon^* = (\epsilon b_0)^{1/3} V_0^{-1} \quad N^* = N b_0 / V_0$$

respectively.

#### 4. VORTEX DECAY

##### 4.1 Review of Recent Studies

Wake vortices rarely last longer than several minutes (e.g., Campbell et al 1997, Sarpkaya 1998), implying that significant decay processes are at work. Mechanisms for vortex decay include: 1) molecular diffusion, 2) turbulence diffusion, 3) stratification, 4) ground interaction and 5) three-dimensional instabilities. Typical Reynolds numbers ( $Re = \Gamma_0/\nu$ ) for atmospheric wake vortices are on the order of  $10^7$ , implying that molecular diffusion is inconsequential. Three-dimensional instabilities such as Crow instability (Crow and Bate 1976) have long been known to induce rapid decay and shorten the lifespan of wake vortices. Crow instability occurs when two trailing vortices undergo a sinusoidal instability causing the two vortices to link forming crude ring structures (see Fig. 2). The onset of Crow instability is most susceptible to perturbations with scales of about  $8.6 b_0$  (Crow 1970). The onset of vortex linking from Crow instability appears to depend upon the ambient turbulence intensity (Sarpkaya and Daly 1987, Han et al 2000a) and is independent of the ambient stratification (Switzer and Proctor 2000). Sarpkaya (1998) has recently reanalyzed Crow and Bate's (1976) analytical solution for the time to link. He found the link time as (in nondimensional units):

$$\begin{aligned} T &= 0.8039\epsilon^{*3/4} \quad \text{for } \epsilon^* > 0.2535 \\ T^{1/4} \text{Exp}(-0.7T) &= \epsilon^* \quad \text{for } 0.0121 < \epsilon^* < 0.2535 \\ T &= -180\epsilon^* + 9.18 \quad \text{for } 0.001 < \epsilon^* < 0.0121 \\ T &= -9 \quad \text{for } \epsilon^* < 0.001 \end{aligned}$$

This relationship, which is plotted in Fig. 3, seems to be confirmed by TASS LES data. Other three-dimensional instabilities have been known to influence vortex decay as well. Recently, Delisi and Robins (2000) have investigated a short-scale instability with a scale of about one  $b_0$ , which is associated with wake vortices in a stratified fluid.

Other decay mechanisms include atmospheric turbulence and turbulence generated by the aircraft. The later is probably of secondary importance, but guarantees some minimal decay when atmospheric conditions are near laminar. Although turbulence is strongly damped by rotation near the vortex core, turbulence can be amplified outside the core by the mutual strain of the vortex pair. Turbulence with scales smaller than the vortex separation can lead to the steady removal of vorticity, thus allowing the vortices to weaken over time. The effect of vortex decay via ambient turbulence was isolated from other decay mechanisms in the

TASS LES simulations of Han et al (2000b). In this study, a wake vortex pair was exposed to isotropic turbulence within a neutrally-stratified environment, and a short domain length was chosen to suppress the development of Crow instability. This study showed that vortex decay from turbulence diffusion could be related directly to the nondimensional eddy dissipation rate.

Wake vortex decay rates also can be strongly enhanced by the viscous interaction with the ground (Proctor and Han 1999, Proctor et al 2000). In Proctor et al (2000), a model for the vortex decay rate was formulated and vortex linking with the ground was examined. As with the case of Crow instability, linking with the ground was found to be a function of the nondimensional turbulence intensity. However, vortex decay rates were found to be independent of ambient turbulence following vortex rebound.

In the past, controversy has surrounded the decay process, such as do vortices decay at a continuous and predictable rate under the influence of the various decay mechanisms, or do they decay very little until sudden destruction is initiated by some stochastic event (see Spalart 1998). Our results tend to support the former, but with enhanced levels of decay from three-dimensional instability processes. Moreover, the onset of the instabilities seems to be predictable functions of the atmospheric state.

##### 4.2 Additional TASS Results

Vortex decay is further examined by conducting additional experiments with TASS. As in previous investigations (e.g., Switzer and Proctor 2000), a  $1.5\text{ m}$  grid size is assumed in the lateral and vertical directions, and a  $2\text{ m}$  grid size in the longitudinal direction. The domain is assumed to be fully periodic and is initialized with a Kolmogorov spectrum of ambient turbulence. Results from these runs are shown in Figs. 2-11. As in previous experiments with TASS, the linking times associated with Crow instability are independent of stratification and dependent upon the ambient turbulence intensity.

Figure 4 shows the decay of average circulation as a function of three different ambient turbulence intensities. Note that rapid decay occurs during linking with the destruction of the vortex occurring soon after the linking time. For empirical models to correctly forecast the vortex sink rate and trajectory, it is important to predict the effective circulation at the vortex separation distance ( $b$ ) – that is the circulation based on the vortex sink rate,  $\Gamma_b = 2\pi\gamma b$ . This effective circulation decays very weakly until the linking time, after which it decays rapidly (Fig. 5). However, the decay rate of this parameter may differ from the decay rate of average circulation that is associated with the vortex hazard.

Figures 6-8 show circulation decay curves at various levels of ambient stratification. The onset of rapid decay for stratified fluids occurs *prior* to the vortex linking times. This onset of sudden decay is associated with short-scale instability (Delisi and Robins 2000) as can be seen in Fig. 10. Note from Fig. 6, that the characteristic of the rate of decay from short-scale instability is similar to that during Crow instability (cf. Fig 4). However, the effective circulation at radius  $b$  decays in a more continuous fashion. This may be due to the continuous baroclinic production of countersign vorticity within the

periphery of the vortex, while short-scale instability must grow before becoming a significant dissipation mechanism with the vortex core.

The onset time for rapid decay of the average circulation can be modeled as:

$$T_s = - [1.27 \ln(\epsilon^*) - 0.57] \exp(-1.15 N^*), \text{ for } \epsilon^* < 0.25.$$

This relationship fits the TASS LES data extremely well (Fig. 9) and considers long as well as short wave instability. For comparison with vortex time to link, the curve for the above relationship is plotted in Fig 3 for neutral stratification.

## 5. SUMMARY AND CONCLUDING REMARKS

Our results tend to support the hypothesis for a continuous and predictable rate of vortex decay, but with enhanced levels following the development of three-dimensional instability processes. Moreover, the onset of the instabilities seems to be predictable functions of the atmospheric state. A new relationship for the onset of sudden decay is proposed which can be adapted into wake vortex prediction models for aircraft spacing.

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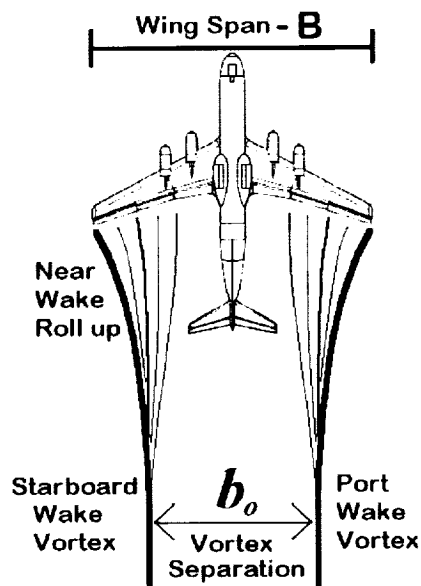


Figure 1. Aircraft wake vortices in relation to generating aircraft (viewed from below).

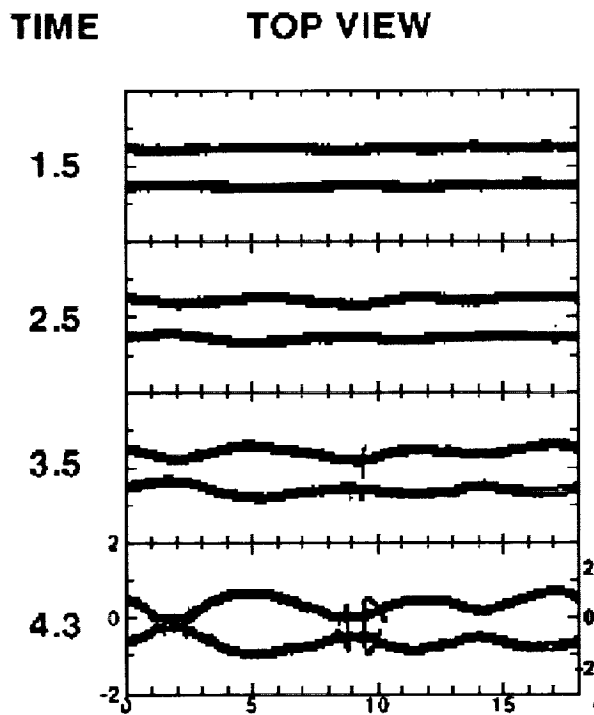


Figure 2. Top view of time evolution of wake vortex from TASS results for case with  $N^*=0$  and  $\epsilon^*=0.07$ . The last time corresponds to the theoretical time to link. The nondimensional time is labeled to the left of each row. The domain axial dimensions are shown at the bottom row and are given in units normalized by  $b_o$ .

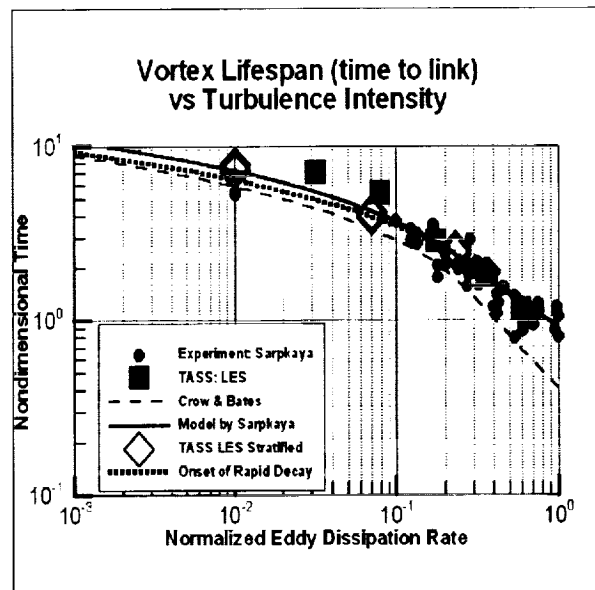


Figure 3. Nondimensional time to link vs nondimensional eddy dissipation rate. Onset time for rapid decay at  $N^*=0$  also shown.

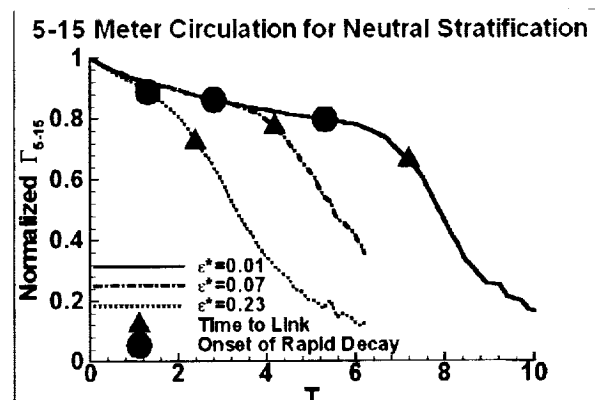


Figure 4. TASS nondimensional average circulation vs nondimensional time for neutral stratification.

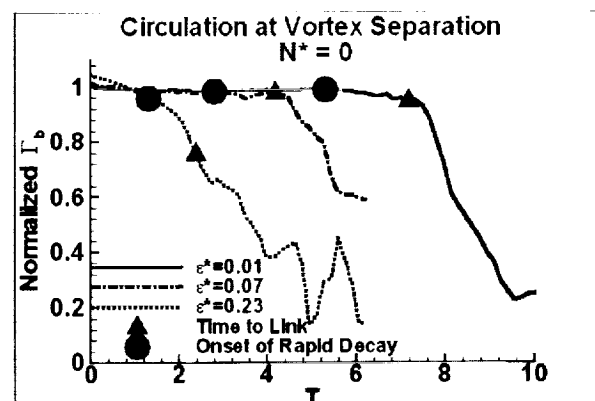
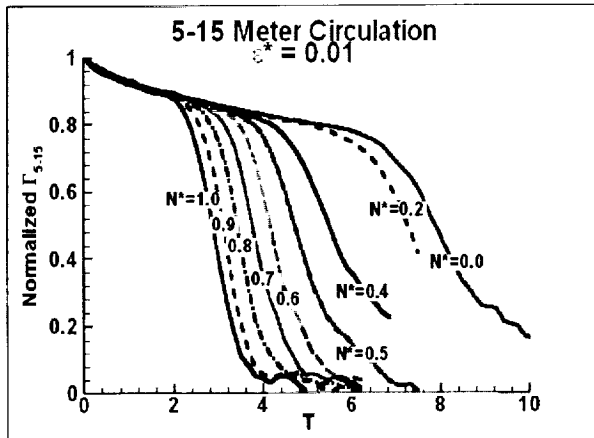
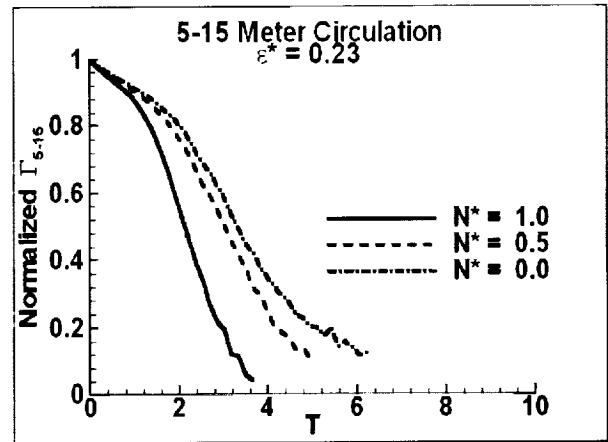


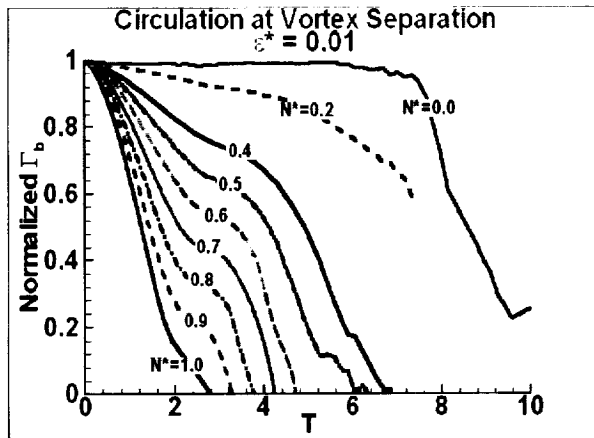
Figure 5. Same as Fig. 4, but for nondimensional circulation at radius of vortex separation.



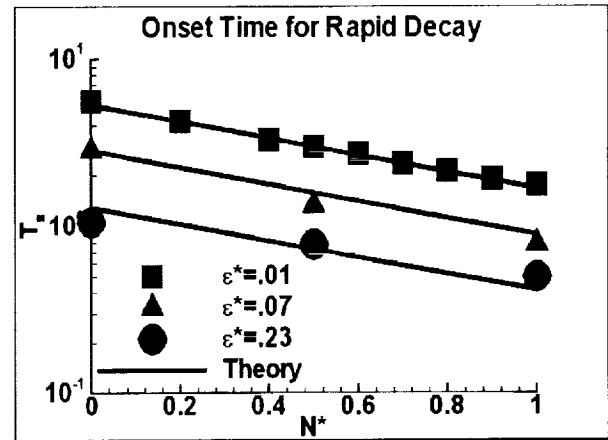
**Figure 6.** Same as Fig. 4, but for  $\epsilon^*=0.01$  with different levels of nondimensional  $N$ .



**Figure 8.** Same as Fig. 6, but for a moderate intensity of ambient turbulence.



**Figure 7.** Same as Fig. 5, but for different levels of nondimensional  $N$ .



**Figure 9.** Onset of rapid decay vs nondimensional  $N$  and nondimensional eddy dissipation. Relationship from paper plotted as solid line.

$$N^* = 1.0 - \epsilon^* = 0.01$$

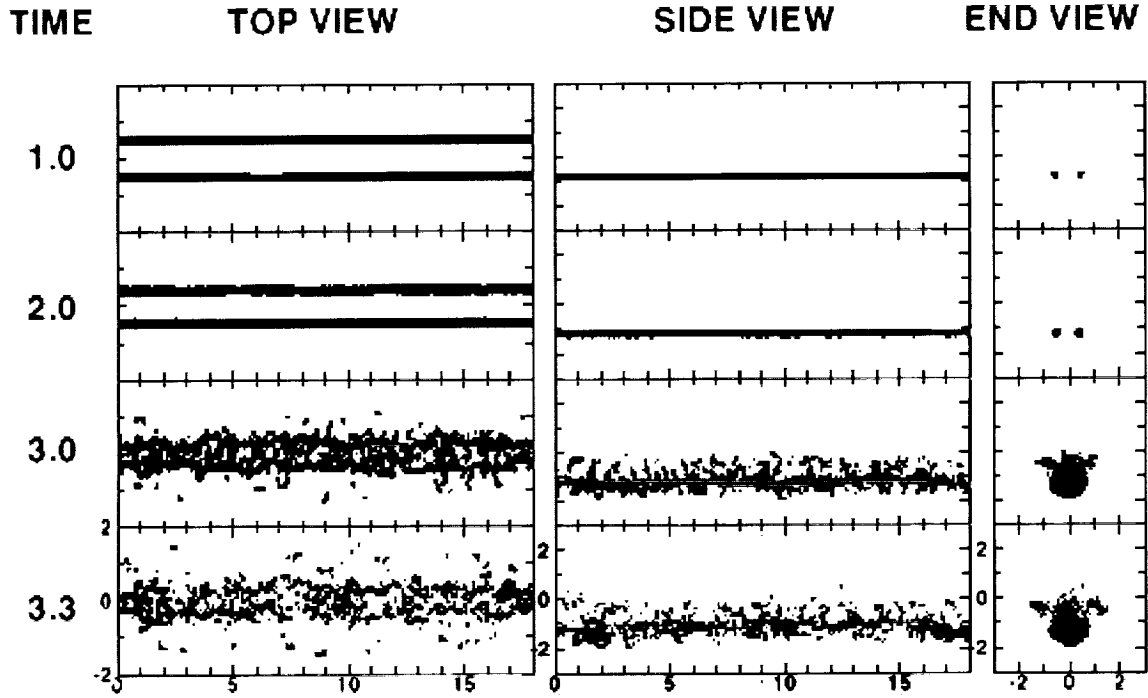


Figure 10. Same as Fig. 2, but for  $N^*=1.0$  and  $\epsilon^*=0.01$ . The last time is shortly before the vortex dissipates.

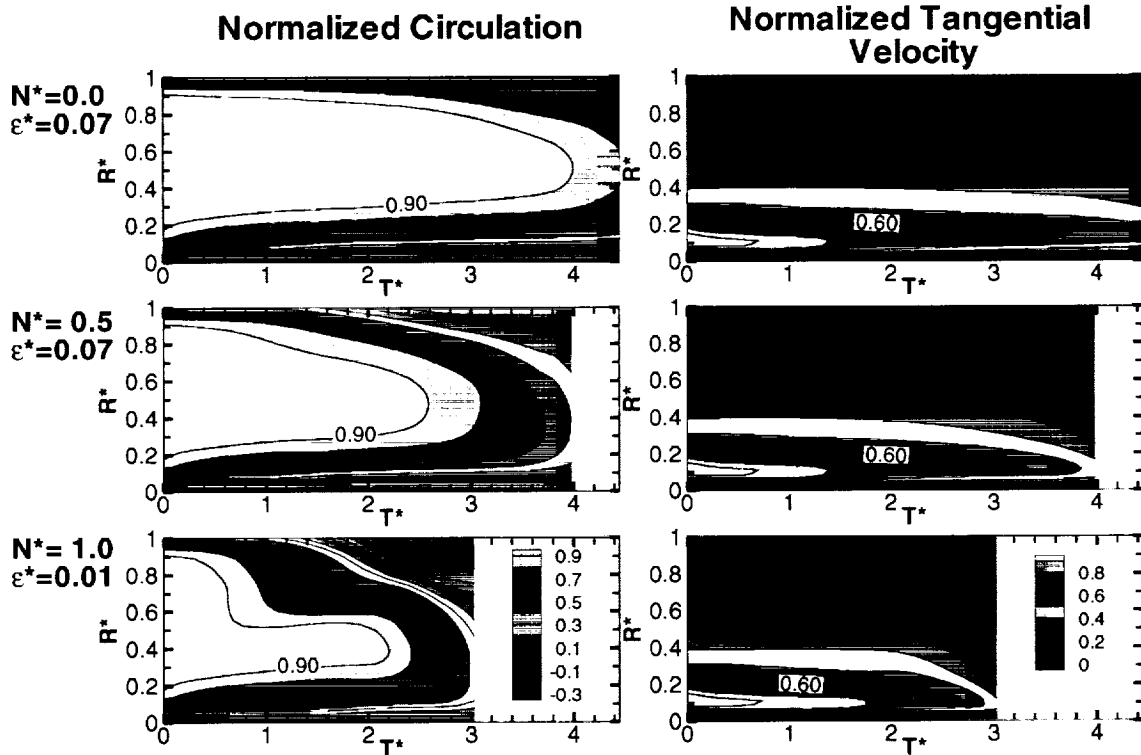


Figure 11. Time evolution of axially averaged radial profile of circulation and tangential velocity for  $N^*$  of 0.0, 0.5 and 1.0 at the weak turbulence intensity levels. Radial ( $R^*$ ) distance is normalized by  $b_0$ . Circulation and tangential velocity are normalized by the peak value at  $T^*=0$ .

**Cape Girardeau Model Data**  
Provided by Fred Proctor of NASA LaRC  
Y = -700

